EXHIBIT A

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EXHIBIT B

UBS

Global Equity Research

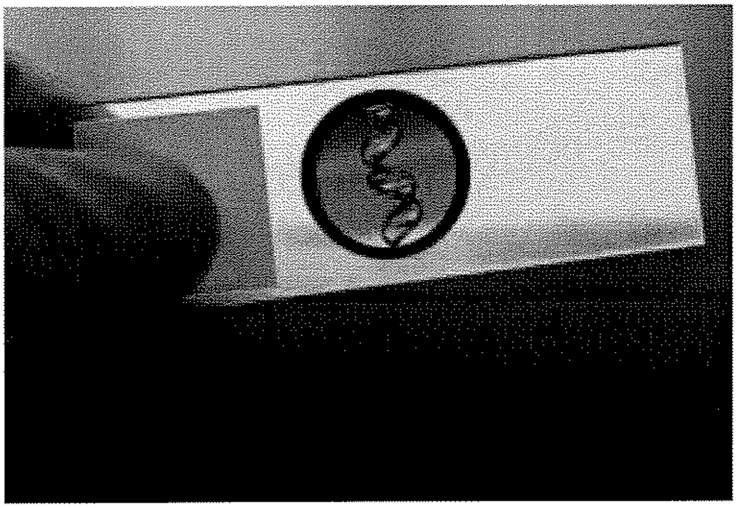
Americas

Biotechnology

Sector Comment

UBS Investment Research

Q-Series®: The DNA Microarray Market



Mega Markets for Microarrays?

23 January 2006

www.ubs.com/investmentresearch

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- **What is the DNA microarray market opportunity?**

DNA microarray technology revolutionized whole genome analysis. Most microarrays are used for gene expression studies, but this market is mature. As new applications in genotyping and molecular diagnostics emerge, key questions for life sciences investors are whether these new markets are real and how fast they can grow.

- **UBS market model used to forecast microarray demand**

To address these questions, we built a proprietary model for the microarray market and project demand through 2010. We used interviews with experts and industry participants to define the existing market, and forecast growth based on customer surveys and a review of R&D trends.

- **The market is real, but could be smaller than expected**

In our view, the microarray opportunity is real, albeit smaller than some earlier forecasts. We estimate the total microarray market will grow from ~$875 million in 2005 to ~$1.6 billion by 2010 (13% CAGR). We see gene expression stabilizing at ~7% growth, while we believe genotyping could be a ~$500 million market. However, we are cautious on molecular diagnostics.

- **Investment implications**

Affymetrix (Neutral 2) is the market leader, but execution problems have hurt performance; we present three scenarios and a valuation model for AFFX shares. We also expect Illumina, Agilent, Applied Biosystems, and GE Healthcare to benefit from the DNA microarray market opportunity.

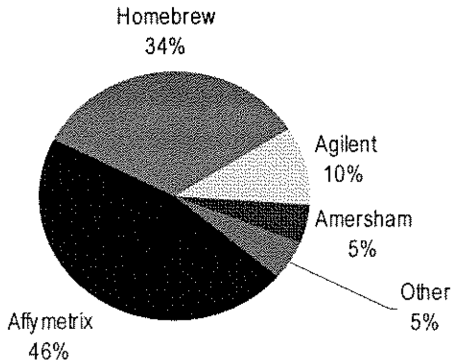
This report has been prepared by UBS Securities LLC

Table 1: Selected Microarray Providers

Company	Manufacturing Approach	Probe Length	Detection Method
Affymetrix	Photolithography	25-mer	Fluorescence
Agilent	Inkjet	60-mer	Fluorescence
GE Healthcare/Amersham	Inkjet	30-mer	Fluorescence
ABI	Non-contact spotting	60-mer	Chemiluminescence
Combimatrix	Microelectronic	Up to 35-mer	Fluorescence
Nanogen	Microelectronic	--	Fluorescence
Nimblegen	"Maskless" Photolithography	Up to 70-mer	Fluorescence
Phalanx	Inkjet	60-mer	Fluorescence

Source: UBS and corporate reports

Chart 4: Gene Expression Market Share Estimates, 2004



Note: "Other" includes Applied Biosystems, Illumina, Combimatrix, Nanogen, and Nimblegen.
Source: UBS estimates and corporate reports

We estimate that the gene expression market is growing at 6-9% per year

Based on our user surveys, interviews with experts and industry contacts, and commercially available market research data, we estimate the "core" gene expression market is growing at 6-9% per year. We believe the decline from the double-digit growth rates seen prior to 2004 is due to a drop in the price per array and a decline in array-based drug target discovery efforts at Big Pharma and Biotech. Looking forward, we believe the introduction of new transcriptome applications, increased focused on biomarker discovery, clinical research-related applications, and high-throughput instrumentation will help reinvigorate gene expression market growth.

We believe that transcriptome research will be a key market growth driver

Specifically, beyond the core gene expression market, advances in high-density microarray production combined with better genomic information are enabling new avenues of transcriptome research outside the known and annotated genes and coding regions. New GeneChip Tiling Arrays by Affymetrix have been designed to interrogate the genome at regular intervals, including both the coding and non-coding regions thought to be involved in gene regulation. Using tiling arrays, one can begin to examine sites of DNA methylation, transcription

EXHIBIT C

INFINIUM

Company Report

March 30, 2006

Affymetrix Inc.

Not out of the woods yet

Affymetrix engages in the development, manufacture, sale and service of microarray platforms in the life science research and clinical diagnostics markets. The company holds leading market share in both the gene expression and SNP (single nucleotide polymorphism) genotyping markets.

Valuation & Recommendation

We are initiating coverage on Affymetrix with an Underperform rating and a 12-month price target of $26.00, based on a DCF valuation. The key question for Affymetrix at this point is whether all the bad news has been priced in. Despite the company losing half of its market value over the past 6 months, we believe there is still risk on the downside.

No room for weakness implied in guidance

With the gene expression market maturing, the key driver of growth for Affymetrix is the emerging SNP genotyping market. We estimate the company will need to grow 50%+ in its DNA Analysis (SNP genotyping) business (~25% of 2005 revenues) in order to hit its growth target of 15% for the overall company. We see risks that Affymetrix may not reach revenue targets. Our proprietary survey sampling 9 Directors (18% response rate) from 50 of the largest academic genotyping centers in the world suggests that Illumina is well positioned to take share from Affymetrix in 2006.

Cautious long-term outlook in WGA SNP genotyping

The whole-genome association (WGA) SNP genotyping market (roughly 2/3 of the overall $250 mln market) is ramping quickly in the near-term, but will slow down sooner than anticipated. In the near-term, the market will grow very rapidly in 2006 (>50%) as a large wave of WGA studies take place. However, we believe that demand for these large-scale studies will flatten in 2007 – leading us to forecast 5-year CAGR of 20% until 2010.

Early days for molecular diagnostics

At this time, we believe the market opportunity for array-based molecular diagnostics is 2 to 3 years away. Key reasons to support our cautious view include slow ramp in the GeneChip 3000Dx unit placements and limited uptake of Roche's CYP450 AmpliChip.

Report priced at the close on March 29, 2006.

Please see risks on pg. 14 and important disclosures & legal disclaimers beginning on pg. 27

Analyst

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Scientific Instruments

United States

Initiating coverage with a rating of:

UNDERPERFORM

$31.52

12-month target	**$26.00**
Required rate of return	**10.0%**
Expected rate of return	**-17.5%**
PE (2007e EPS)	**29.4**
Codes	
NASDAQ	AFFX
Key Data: in US$	
Current fiscal year-end	December
Shares OS Basic (mln)	65.9
Shares OS FD (mln)	72.3
Market cap ($mln)	2,078.3
Net debt (cash) ($mln)	164.9
Enterprise value ($mln)	1,913.3
Major Shareholder:	
Fidelity, AXA	14.4%, 13.5%
Other trading data	
52-week high ($)	59.73
52-week low ($)	29.80
Avg daily volume 3 month (mln)	1.73
Shares short – Feb 2006 (mln)	8.4
Historical 30 day volatility	47.2%
Implied volatility	48.4%

Highlights	2004	2005	2006E	2007E
Revenues	346.0	367.6	408.5	459.1
EBIT	60.6	66.8	56.0	93.3
Net income	48.5	66.9	48.2	76.7
EPS	0.75	0.97	0.67	1.04

EPS (IS, US$)	Infinium current	Consensus current	Actual prior year
Q1E	0.04	0.05	0.24
Q2E	0.08	0.07	0.12
Q3E	0.12	0.10	0.13
Q4E	0.22	0.25	0.48
FY 06E	0.45	0.46	0.97
FY 07E	0.67	0.83	na

Infinium Capital research is available electronically on Thomson First Call, Reuters and Bloomberg
Consensus estimates from Thomson First Call; trading data from Bloomberg; estimates from Infinium
Issued by Infinium Capital.

Market Overview

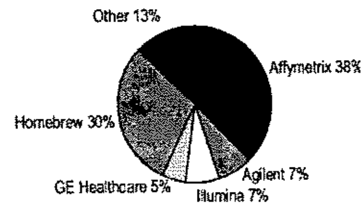
Affymetrix engages in the development, manufacture, sale and service of microarray platforms in the life science research and clinical diagnostics markets. Affymetrix is the market leader in the $800 mln microarray market. The company primarily competes in the gene expression (RNA Analysis) and SNP (single nucleotide polymorphism) genotyping (DNA Analysis) markets.

Microarray Market

We estimate the microarray market to be roughly $800 mln in 2005 split 70:30 between the commercial and homebrew segments. Microarrays are used most often to study gene expression (estimated 80% of the market). This is a mature market with estimated growth rates ranging from 6% to 10%.

Homebrew technologies give researchers the flexibility to self-print custom content microarrays at a fraction of the cost of commercial microarrays. It is estimated roughly $100 mln (of the $800 mln) of the market will remain homebrew due to its cost and flexibility to customize content. While the homebrew technologies have a cost advantage, over time, the commercial segment is expected to grow faster as customers shift towards more standardized platforms.

Worldwide Microarray Market ($800 mln)



Source:

Microarray Market (2005 estimate of $800 mln)

Category	Commercial	Homebrew
Description	Off-the-shelf microarrays pre-fabricated using inkjet-based, bead or photolithographic techniques	Self-printed microarrays using robotic arrayers and analyzed through a generic scanner
Size	$550 mln	$250 mln
Leading companies	Affymetrix, Agilent, Illumina	Agilent, PerkinElmer, Molecular Devices, Illumina
Outlook	Pricing pressures in catalog arrays; shift towards custom arrays	Will decline going forward as customers shift towards commercial platforms

Source: Infinium estimates

Affymetrix dominates the commercial segment

Affymetrix has an estimated 38% share of the overall microarray market and 54% share amongst the commercial suppliers. The first Affymetrix GeneChip® microarray was marketed over 10 years ago in 1994. Feature size has decreased from 100 to 5 microns thereby increasing the number of features from 16K features versus 6.0 mln today enabling each chip to encompass much more genomic content. Since then, the company has established itself as the clear market leader - with an installed base of 1,375 platforms at year end 2005.

First-to-market Affymetrix

Year	Company
1994	Affymetrix
2000	Agilent
	GE Healthcare
2003	Illumina
2004	Applied Biosystems

Source: Infinium

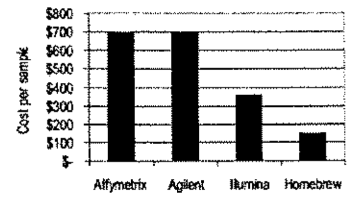
Agilent Technologies (A, $39.36) – a competing microarray platform launched in 2000 quickly gained a foothold by offering an open, flexible platform compatible with existing equipment used by the homebrew market. By year end 2003, Agilent's installed base grew to 400 (versus Affymetrix at 975). In addition,

Market Overview

Agilent's inkjet manufacturing process offers more flexibility than photolithographic techniques employed by Affymetrix – which is well suited for catalog arrays and long production runs. The U.S. FDA recently selected Agilent's dual-mode platform as part of the Microarray Quality Control (MAQC) project – designed to provide quality control guidance to the microarray industry.

In 2003, Illumina launched whole genome gene expression arrays compatible with its BeadStation 500GX – used mainly for SNP genotyping applications. The company's catalog arrays for whole genome analysis are priced to undercut Affymetrix and Agilent. However, price points have yet to come down to be in-line with homebrew arrays (as illustrated in the chart at right). Smaller companies such as CombiMatrix (CBMX, $2.47) and NimbleGen are also well regarded by academic researchers in providing custom arrays.

Pricing for gene expression arrays



Source: Infinium

Emerging applications for microarrays include SNP genotyping, chromatin immunoprecipitation (chIP), and comparative genomic hybridization (CGH). The array-based SNP genotyping market is the fastest growing segment with Affymetrix and Illumina dominating the field. Other applications in chIP and CGH are just recently emerging – but growing 2 to 4 times faster than the gene expression market. Agilent believes this market could represent 10% of the overall market by 2007. The table below highlights the key competitors in the commercial microarray market.

Commercial Microarray Suppliers

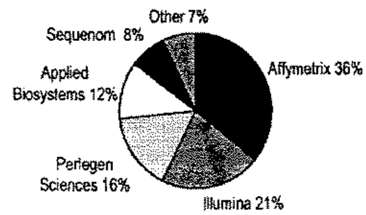
	FY2005e Sales ($ mln)	Platform	Installed base	Open/closed system	Manufacturing	Detection	Probe Length	Applications*
Affymetrix	303.0	GeneChip® System 300	1,375	Closed	Photolithography	Fluorescence	25-mer	GE, GT, E
Agilent	60.0	Dual-mode Gene Expression Microarray Platform	500	Open	Inkjet	Fluorescence	60-mer	GE, E
Illumina	58.0	BeadStation 500	126	Open/closed	Beads	Fluorescence	50-mer	GE, GT
GE	40.0	CodeLink Bioarray	120	Open	3-D aqueous gel matrix	Fluorescence	30-mer	GE
ABI	16.0	1700 Chemiluminescence Microarray Analyzer	Unknown	Closed	Non-contact spotting	Chemiluminescence	60-mer	GE

Note: * GE – gene expression, GT – genotyping, E – emerging applications (sequencing, chIP, CGH)
Source: Infinium estimates

SNP Genotyping Market

Commercial suppliers in this market provide instrument platforms and consumables that enable researchers to determine variations in DNA called SNPs (single nucleotide polymorphisms). SNPs are common, single base variations that occur in approximately 1 of every 1000 bases throughout the genome. It is estimated that the human genome contains between six and ten million SNPs. Due to their abundance and relative stability, SNPs are useful biological markers to study disease susceptibility and/or adverse drug reactions.

SNP Genotyping Market (2005e $250 mln)



Source: Infinium

Technology platforms are defined by the application, the level of multiplexing (number of SNPs analyzed) and throughput (number of processed samples each day). We broadly divide the market between:

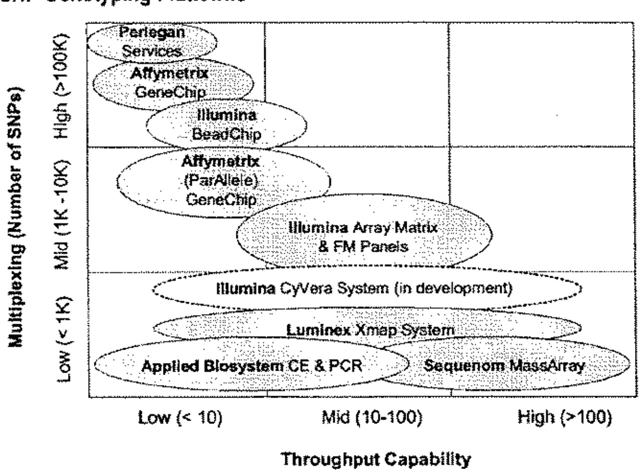
1) **Whole genome association (WGA)** studies (roughly 2/3 of the market) scan SNPs across the genome and require high levels of multiplexing (>100K).

2) **Targeted genotyping (TG)** studies (roughly 1/3 of the market) focus on specific candidate genes or regions and require low-to-mid levels of multiplexing (1 to 20,000 SNPs).

Whole genome association versus targeted genotyping studies require different levels of multiplexing

Illumina, Affymetrix, and Perlegen Sciences (25% owned by Affymetrix) dominate the WGA high-multiplex genotyping market. Perlegen is not an instrument provider but is the market leader in the outsourced SNP genotyping services market. Established technologies adapted from sequencing, gene expression, and flow cytometry compete in the low-to-mid multiplex genotyping market. Please refer to Appendix 1 for a detailed comparison of the various platforms.

Overview of SNP Genotyping Platforms



Source: Infinium

EXHIBIT D

e Methods with Simulation

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Discrete Choice Methods with Simulation

Kenneth E. Train

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CAMBRIDGE
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3.3.2. Substitution Patterns

When the attributes of one alternative improve (e.g., its price drops), the probability of its being chosen rises. Some of the people who would have chosen other alternatives under the original attributes now choose this alternative instead. Since probabilities sum to one over alternatives, an increase in the probability of one alternative necessarily means a decrease in probability for other alternatives. The pattern of substitution among alternatives has important implications in many situations. For example, when a cell-phone manufacturer launches a new product with extra features, the firm is vitally interested in knowing the extent to which the new product will draw customers away from its other cell phones rather than from competitors' phones, since the firm makes more profit from the latter than from the former. Also, as we will see, the pattern of substitution affects the demand for a product and the change in demand when attributes change. Substitution patterns are therefore important even when the researcher is only interested in market share without being concerned about where the share comes from.

The logit model implies a certain pattern of substitution across alternatives. If substitution actually occurs in this way given the researcher's specification of representative utility, then the logit model is appropriate. However, to allow for more general patterns of substitution and to investigate which pattern is most accurate, more flexible models are needed. The issue can be seen in either of two ways, as a restriction on the ratios of probabilities and/or as a restriction on the cross-elasticities of probabilities. We present each way of characterizing the issue in the following discussion.

The Property of Independence from Irrelevant Alternatives

For any two alternatives i and k, the ratio of the logit probabilities is

$$\frac{P_{ni}}{P_{nk}} = \frac{e^{V_{ni}} / \sum_j e^{V_{nj}}}{e^{V_{nk}} / \sum_j e^{V_{nj}}} = \frac{e^{V_{ni}}}{e^{V_{nk}}} = e^{V_{ni} - V_{nk}}.$$

This ratio does not depend on any alternatives other than i and k. That is, the relative odds of choosing i over k are the same no matter what other

alternatives are available or what the attributes of the other alternatives are. Since the ratio is independent from alternatives other than i and k, it is said to be independent from *irrelevant* alternatives. The logit model exhibits this *independence from irrelevant alternatives*, or IIA.

In many settings, choice probabilities that exhibit IIA provide an accurate representation of reality. In fact, Luce (1959) considered IIA to be a property of appropriately specified choice probabilities. He derived the logit model directly from an assumption that choice probabilities exhibit IIA, rather than (as we have done) derive the logit formula from an assumption about the distribution of unobserved utility and then observe that IIA is a resulting property.

While the IIA property is realistic in some choice situations, it is clearly inappropriate in others, as first pointed out by Chipman (1960) and Debreu (1960). Consider the famous red-bus–blue-bus problem. A traveler has a choice of going to work by car or taking a blue bus. For simplicity assume that the representative utility of the two modes are the same, such that the choice probabilities are equal: $P_c = P_{bb} = \frac{1}{2}$, where c is car and bb is blue bus. In this case, the ratio of probabilities is one: $P_c/P_{bb} = 1$.

Now suppose that a red bus is introduced and that the traveler considers the red bus to be exactly like the blue bus. The probability that the traveler will take the red bus is therefore the same as for the blue bus, so that the ratio of their probabilities is one: $P_{rb}/P_{bb} = 1$. However, in the logit model the ratio P_c/P_{bb} is the same whether or not another alternative, in this case the red bus, exists. This ratio therefore remains at one. The only probabilities for which $P_c/P_{bb} = 1$ and $P_{rb}/P_{bb} = 1$ are $P_c = P_{bb} = P_{rb} = \frac{1}{3}$, which are the probabilities that the logit model predicts.

In real life, however, we would expect the probability of taking a car to remain the same when a new bus is introduced that is exactly the same as the old bus. We would also expect the original probability of taking bus to be split between the two buses after the second one is introduced. That is, we would expect $P_c = \frac{1}{2}$ and $P_{bb} = P_{rb} = \frac{1}{4}$. In this case, the logit model, because of its IIA property, overestimates the probability of taking either of the buses and underestimates the probability of taking a car. The ratio of probabilities of car and blue bus, P_c/P_{bb}, actually changes with the introduction of the red bus, rather than remaining constant as required by the logit model.

This example is rather stark and unlikely to be encountered in the real world. However, the same kind of misprediction arises with logit models whenever the ratio of probabilities for two alternatives changes with the introduction or change of another alternative. For example, suppose a new transit mode is added that is similar to, but not exactly like, the existing modes, such as an express bus along a line that already has

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standard bus service. This new mode might be expected to reduce the probability of regular bus by a greater proportion than it reduces the probability of car, so that ratio of probabilities for car and regular bus does not remain constant. The logit model would overpredict demand for the two bus modes in this situation. Other examples are given by, for example, Ortuzar (1983) and Brownstone and Train (1999).

Proportional Substitution

The same issue can be expressed in terms of the cross-elasticities of logit probabilities. Let us consider changing an attribute of alternative j. We want to know the effect of this change on the probabilities for all the *other* alternatives. Section 3.6 derives the formula for the elasticity of P_{ni} with respect to a variable that enters the representative utility of alternative j:

$$E_{iz_{nj}} = -\beta_z z_{nj} P_{nj},$$

where z_{nj} is the attribute of alternative j as faced by person n and β_z is its coefficient (or, if the variable enters representative utility nonlinearly, then β_z is the derivative of V_{nj} with respect to z_{nj}).

This cross-elasticity is the same for all i : i does not enter the formula. An improvement in the attributes of an alternative reduces the probabilities for all the other alternatives by the same percentage. If one alternative's probability drops by ten percent, then all the other alternatives' probabilities also drop by ten percent (except of course the alternative whose attribute changed; its probability rises due to the improvement). A way of stating this phenomenon succinctly is that an improvement in one alternative draws proportionately from the other alternatives. Similarly, for a decrease in the representative utility of an alternative, the probabilities for all other alternatives rise by the same percentage.

This pattern of substitution, which can be called *proportionate shifting*, is a manifestation of the IIA property. The ratio of probabilities for alternatives i and k stays constant when an attribute of alternative j changes only if the two probabilities change by the same proportion. With superscript 0 denoting probabilities before the change and 1 after, the IIA property requires that

$$\frac{P^1_{ni}}{P^1_{nk}} = \frac{P^0_{ni}}{P^0_{nk}}$$

when an attribute of alternative j changes. This equality can only be maintained if each probability changes by the same proportion: $P^1_{ni} = \lambda P^0_{ni}$ and $P^1_{nk} = \lambda P^0_{nk}$, where both λ's are the same.

Proportionate substitution can be realistic for some situations, in which case the logit model is appropriate. In many settings, however, other patterns of substitution can be expected, and imposing proportionate substitution through the logit model can lead to unrealistic forecasts. Consider a situation that is important to the California Energy Commission (CEC), which has the responsibility of investigating policies to promote energy efficient vehicles in California and reducing the state's reliance on gasoline for cars. Suppose for the sake of illustration that there are three kinds of vehicles: large gas cars, small gas cars, and small electric cars. Suppose also that under current conditions the probabilities that a household will choose each of these vehicles are .66, .33, and .01, respectively. The CEC is interested in knowing the impact of subsidizing the electric cars. Suppose the subsidy is sufficient to raise the probability for the electric car from .01 to .10. By the logit model, the probability for each of the gas cars would be predicted to drop by the same percentage. The probability for large gas car would drop by ten percent, from .66 to .60, and that for the small gas car would drop by the same ten percent, from .33 to .30. In terms of absolute numbers, the increased probability for the small electric car (.09) is predicted by the logit model to come twice as much from large gas cars (.06) as from small gas cars (0.03).

This pattern of substitution is clearly unrealistic. Since the electric car is small, subsidizing it can be expected to draw more from small gas cars than from large gas cars. In terms of cross-elasticities, we would expect the cross-elasticity for small gas cars with respect to an improvement in small electric cars to be higher than that for large gas cars. This difference is important in the CEC's policy analysis. The logit model will overpredict the gas savings that result from the subsidy, since it overpredicts the substitution away from large gas cars (the "gas guzzlers") and underpredicts the substitution away from small "gas-sipper" cars. From a policy perspective, this misprediction can be critical, causing a subsidy program to seem more beneficial than it actually is. This is the reason that the CEC uses models that are more general than logit to represent substitution across vehicles. The nested logit, probit, and mixed logit models of Chapters 4–6 provide viable options for the researcher.

Advantages of IIA

As just discussed, the IIA property of logit can be unrealis[tic] in many settings. However, when IIA reflects reality (or an adequ[ate] approximation to reality), considerable advantages are gained by its [em]ployment. First, because of the IIA, it is possible to estimate mo[del]

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International Journal of the
Economics of *Business, Vol. 7, No. 2, 2000, pp. 213–227*

Lost Profits from Patent Infringement: The Simulation Approach

GREGORY J. WERDEN, LUKE M. FROEB and, JAMES LANGENFELD

ABSTRACT *A patent owner is entitled to recover any additional profits that would have been earned but for infringement. This paper suggests the use of an adaptation of merger simulation to assess lost profits in patent infringement cases. A model of the industry with infringement is calibrated to observed prices and quantities and estimated demand elasticities. Lost profits are then estimated by calculating a new equilibrium without the infringing product(s).*

Key words: Simulation; Patents; Damages.

JEL classification: K29, L13, O34.

1. Introduction

Once a patent owner establishes infringement, the Patent Act requires the award of 'damages adequate to compensate for the infringement.'[1] Damages are assessed as lost profits, reasonable royalties, or a combination of the two. In both frequency and amount, awards of lost profits have become very significant in recent years (see Davis and Kedrowski, 1999). To recover lost profits for patent infringement, the patent owner must show a reasonable probability that it would have received the additional profits 'but for' the infringement. The courts have prescribed no one particular method by which the patent owner must meet this burden.[2] Precision in the estimation of damages is not required, and uncertainty tends to be resolved against those found to have infringed.

In considering whether to award lost profits, patent infringement cases devote a great deal of attention to an analog to antitrust market delineation. The issue is framed as whether there are 'acceptable' non-infringing substitutes

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for the patentee's product. Patent damage cases create the sort of false dichotomy that is familiar from antitrust cases, treating products determined to be 'in the market' as almost perfect substitutes, and ignoring competition from outside.

If it is determined that there are no non-infringing substitutes for the patentee's product, courts readily infer that the infringer diverted sales from the patentee and award lost profits for the sales diversion. The courts have sometimes noted that competition from an infringer affects prices, and rarely noted that price affects output. The courts do not appear to have brought together the sales diversion, price, and quantity issues in a unified analysis that reconstructs the market equilibrium but for the infringement.

We offer a practical way in which that reconstruction can be done.[3] It is a straightforward adaptation of merger simulation, which has recently become conventional in antitrust.[4] Merger simulation estimates the relevant demand relationships among competing products using price and quantity data, and calibrates a standard economic model of competitive interaction to observed prices and quantities. The calibrated model then predicts the effects of a merger on the prices and quantities of the merging firms and their rivals.

Simulating lost profits from patent infringement is similar to simulating the effects of a merger. Rather than extrapolate from the lower-price, pre-merger equilibrium to the higher-price, post-merger equilibrium, one extrapolates from the lower-price, with-infringement infringement equilibrium to the higher-price, but-for-infringement equilibrium. While the former extrapolation involves internalizing competition among the merging products, the latter extrapolation involves eliminating the infringing product(s). Both simulations predict the effect of the change in structure on the sales and prices of competitors. And both simulations depend on the structure of demand, which is partially estimated and partially assumed. Simulation eliminates the need to draw a bright line between close substitutes and not-so-close substitutes. The competitive effects of all substitutes are considered, in some cases by including the product in the simulation, and in others by allowing substitution to outside goods.

The courts' focus on the infringer's diversion of sales from the patentee has made them reluctant to award damages for lost profits when there are 'acceptable' non-infringing substitutes, since the infringer's sales were almost certainly not entirely diverted from the patentee. Lost profits damages have been awarded under the 'market-share rule' when courts were satisfied that market shares provide a reasonable indication of how the infringer's sales would be reallocated if it were eliminated from the market, i.e. when consumer preferences exhibit the Independence of Irrelevant Alternatives (IIA) property. Whenever it is reasonable to invoke the market-share rule, damages can be better assessed in a simulation based on the highly parsimonious logit demand model, which exhibits the IIA property. We illustrate logit simulation with a hypothetical based on the facts of the case in which the market-share rule was established.

When the market-share rule and the logit model are not reasonable representations of actual demand conditions, simulations based on more flexible demand systems can be used. We illustrate this using nested logit demand in a hypothetical based on the facts of the leading case in which the courts declined to apply the market-share rule. Finally, we illustrate damage simulation when the patent owner and infringer compete in a procurement auction.

2. Market Delineation in Patent Cases: Identifying 'Acceptable' Substitutes

The courts have viewed patent damages issues primarily through the analytic lens of causation, awarding the patentee lost profits associated with sales reasonably shown to have been diverted away by the infringement.[5] The focus on causation derives from the fact that a patentee may lose sales and profits for many reasons other than infringement and should not be compensated by the infringer for such loses. The courts often emphasize that causation is a relatively simple matter with just two suppliers—the patentee and the infringer: 'In a market with only two viable competitors, one may infer that the patentee would have made the infringer's sales or charged higher prices but for the infringing competition.'[6]

The two-supplier market is an abstraction; there are always substitutes for the patented product. Hence, the cases have asked whether available substitutes are 'acceptable,' and 'to prove that there are no acceptable noninfringing substitutes, the patent owner must show either that (1) the purchasers in the market place generally are willing to buy the patented product for its advantages, or (2) the specific purchasers of the infringing product purchased on that basis.'[7] Often, only nearly identical products are deemed 'acceptable.' In one oft-cited case, only wheelbarrows with the particular design features of the patentee's product were deemed 'acceptable substitutes.'[8]

The definition of 'acceptable substitutes' is analogous to market delineation in antitrust cases. In one leading case, the court even framed the issue as whether 'the patent owner and the infringer compete in the same market.'[9] As a general rule, 'the market in patent cases is defined in very narrow terms.'[10] Culbertson and Weinstein (1988), Gould and Langenfeld (1997: 465–66, 474–76), Jarosz and Page (1993: 316–19), Krosin and Kozlowski (1990: 70–72), and Meyer (1991: 1372, 1388–91) have all questioned the narrowness of patent markets and suggested the importation of antitrust market delineation from the Merger Guidelines or the antitrust case law.[11] While that would be a substantial improvement, there would remain problems well known in merger analysis.[12]

The purpose of market delineation is to simplify a complex world, by dividing substitutes into two groups—one in which all products generally are treated as equally good substitutes for each other, and a second consisting of products usually considered irrelevant. The treatment of products in both groups is typically a rough approximation of the real world and often ignores much that is significant. Moreover, so much is likely to be at stake in the line drawing process, that the focus of litigation gets diverted from the issues of ultimate consequence to the line drawing itself. These problems have been noted in antitrust (see Werden, 1997a: 367–71), and merger simulation was developed to avoid arbitrary line drawing and refocus attention on the issues that matter.

3. The Components of Lost Profits Damages

Even when market delineation is trivial because there clearly are no 'acceptable' non-infringing substitutes for the patentee's product, several causation issues may remain. As the courts have recognized, the patentee's lost sales may be less than the infringer's sales because the products are differentiated in some important way. More importantly, one would always expect some price effect from competition between the patentee and the infringer, since duopoly nearly always results in a

lower price and higher market quantity than monopoly. Hence, the effect of infringement would never be merely to divert away some of the patentee's sales. The proper assessment of lost profits resulting from infringement should focus not on the infringer's sales, but rather on the price and quantity that would have prevailed but for infringement.

The case law refers to the price effect of infringement as 'price erosion.' Damages from price erosion are awarded in only a small minority of cases (see Davis and Kedrowski, 1999: 15–16), even though price erosion (to some extent) must be a nearly universal phenomenon. Moreover, a lower price implies a higher quantity, other things being equal, so a necessary consequence of price erosion is 'quantity accretion.'[13] As obvious as this point is to economists, it has been recognized by very few courts. A proper assessment of 'damages adequate to compensate for the infringement' must consider sales diversion, price erosion, and quantity accretion, and since they are interrelated, all three must be considered in conjunction.

The failure to account for price erosion surely can be significant. Rampant infringement could drive the patentee's price down to its marginal cost, in which case lost profits damages based solely on sales diversion would be zero. Price erosion can also be modest, in which case damages based solely on sales diversion would be a good approximation of the true lost profits. To get some perspective on the likely significance of price erosion, we borrow an example from Werden *et al.* (1999a), who also present a graphical analysis. Consider a homogeneous product industry with linear demand and constant marginal cost, and assume that infringement results in a symmetric Cournot duopoly. It is easily shown that the patentee's actual lost profits are 20% greater than damages calculated on the basis of sales diversion alone. On the other hand, accounting for price erosion but not quantity accretion, overestimates damages by 60%.

The impacts of price erosion and quantity accretion are less likely to be significant when products are substantially differentiated. The less close a substitute the infringer's product is for the patentee's product, the less is the effect of the infringement on the patentee's sales and price. If the patentee's and infringer's products are distant substitutes, the infringement may result in very little lost profit. There is also another important contrast between homogeneous and differentiated products industries. With homogeneous products, the infringer competes down industry price, causing a movement down the demand curve. With differentiated products, there is a 'variety enhancement' effect; by increasing product variety, the infringer effectively shifts the industry demand curve out. With homogeneous products, the patentee's lost profits are necessarily greater than the infringer's profits (assuming the two have the same costs) because of the price and quantity effects, but with differentiated products, the patentee's lost profits may be significantly less than the infringer's profits.

4. Calculating Lost Profits in the Absence of 'Acceptable' Non-infringing Substitutes

If the infringer is the patentee's only important competitor, the patentee would be a monopolist without the infringement (at least with respect to a residual demand curve). Thus, the patentee would maximize profits by setting price or quantity to equate its price-cost margin with the reciprocal of its demand elasticity. The patentee's elasticity of demand at the monopoly price cannot be measured directly; rather, it must be determined through extrapolation of the demand curve from the

price and quantity with infringement to the price and quantity without infringement. This can be done by first estimating the elasticity of demand for the products of the patentee and infringer together.[14] The estimation yields a value for the elasticity valid with the infringement, and an assumption must be made about the shape of the demand curve to extrapolate to the monopoly price.[15]

There are two approaches to identifying the patentee's marginal cost. Patent damages cases have long relied on accounting data to estimate the patentee's marginal cost in order to determine the lost profit associated with sales diversion, and it is possible to infer marginal costs from a model of the industry with the infringement.[16] To do the latter, relevant demand parameters are first estimated and those estimates are combined with data on observed prices and quantities to calibrate a demand system for patentee and the infringer. The patentee's and infringer's first-order conditions for profit maximization are then solved for the marginal costs that yield as an equilibrium the prices and quantities observed with infringement.

Both of the foregoing approaches may be unworkable unless the patentee's marginal cost is roughly constant in the relevant range,[17] but that is likely to be the case unless there are significant capacity constraints. Patent infringement cases already consider whether the patentee has the capacity to sell the infringer's quantity in determining whether to award lost profits rather than reasonable royalties.

Treatment of the patentee as a monopolist does not mean that this analysis is valid only when the patentee faces no competition other than the infringer. The key assumption on which the analysis is predicated is not that there are no non-infringing substitutes for the patentee product, but rather that the patentee's pricing does not affect pricing strategies for any non-infringing substitutes. Hence, this approach is applicable when the prices of non-infringing substitutes are determined by supply and demand forces independent of the patentee's price and when the non-infringing substitutes form a competitive fringe.

5. The Market-Share Rule

The courts have been wary about awarding lost profits when there are non-infringing substitutes because of difficulties in inferring causation, but some cases have made that inference. Lost profits have been awarded if non-infringing substitutes were considered insignificant or if the courts were satisfied that the patentee's market share was a reasonable basis for determining the extent to which the infringer diverted sales from the patentee rather than non-infringing substitutes.

The 'market-share rule' was established in *State Industries, Inc. v. Mor-Flo Industries, Inc.*,[18] in which State was awarded lost profits for Mor-Flo's infringement, even though the patentee and infringer were not the only producers of 'acceptable' substitutes. Mor-Flo infringed State's patent on a method of insulating water heaters with foam, and foam insulated water heaters are more energy efficient than those insulated with fiberglass. Finding that State's nationwide share of energy-efficient, residential, gas water heaters was 40% during the period of infringement,[19] the courts awarded lost profits damages on 40% of Mor-Flo's sales, with a reasonable royalty awarded on the remainder.

We understand the courts to have found that, if Mor-Flo's infringing product were eliminated from the market, its sales would have been reallocated among the patentee and sellers of non-infringing substitutes in proportion to those firms'

market shares. But as noted by Jarosz and Page (1993: 318, n.22) and Krosin and Kozlowski (1990: 82), this finding supports a calculation somewhat different from that actually performed in *Mor-Flo* and other cases. The court in *Mor-Flo* multiplied the infringer's quantity by State's share of the total market quantity to arrive at an estimate of the sales diversion from State to Mor-Flo. It would have made more sense, however, to multiply Mor-Flo's quantity by State's share of market quantity excluding that sold by Mor-Flo. Mor-Flo's share was about 20% according to data in the district court's opinion. Assuming the figure was exactly 20%, State should have been awarded lost profits on 50%, rather than 40%, of Mor-Flo's sales. We assume hereafter that the 'market-share rule' is not the calculation actually performed in *Mor-Flo*, but rather the one we find more sensible.

The finding we attribute to the court in *Mor-Flo* is a way of stating the IIA property. The conventional, verbal statement of the IIA property is that the odds ratio of any two choices is independent of the other possible choices. The IIA property probably never holds *exactly* in a real-world industry, but it may be a reasonable approximation in many. (Independance of Irrelevant Alternatives)

6. Simulating Lost Profits with Non-Infringing Substitutes: The Logit Model

The IIA property is associated in economics with the logit model, which was originally developed using the IIA as a starting point (see Werden *et al.*, 1996: 86–87). With the logit model, choice probabilities for individual products i take the form:

$$\pi_i = \frac{\exp(\alpha_i - \beta p_i)}{\Sigma_j \exp(\alpha_j - \beta p_j)}$$

where the α_i are choice-specific constants summarizing product preferences, the p_i are the prices of the product, and β is a demand parameter related to elasticity. The own and cross elasticities of demand are easily shown to be

$$\varepsilon_{ii} = -\beta p_i (1 - \pi_i)$$
$$\varepsilon_{ij} = \beta p_j \pi_j.$$

When using the logit model in simulations, it is convenient to group all the relatively unimportant products into a single 'outside good' with choice probability π_0 and a price that is assumed to be constant (and hence can be set equal to zero). Rather than have π_o reflect the actual likelihood of the choice of an outside good, we find it convenient to use it to scale the choice probabilities for the 'inside goods' (i.e. those other than the outside good). This is done by adding a second demand parameter ε, which is the aggregate elasticity of demand for the inside goods. There are two intuitive ways of defining this elasticity, and for both,

$$\varepsilon = -\beta \bar{p} \pi_0.$$

The aggregate elasticity can be defined such that the price changes for the inside goods are equiproportional, in which case $\bar{p}$ is defined as the share-weighted average

price for the inside goods. The aggregate elasticity can also be defined such that the price changes for the inside goods are equal in absolute terms, in which case $\bar{p}$ is defined as the unweighted average price for the inside goods.

When courts find that the IIA property is a reasonable approximation, it is straightforward to use the logit model to estimate lost profits, accounting for not just sales diversion, but also price erosion and quantity accretion. Normally, that would be done by first estimating the two demand parameters with data on actual choices or possibly survey data. The estimated demand parameters along with shares and prices for the inside goods are sufficient to characterize an industry. The simulation proceeds by solving the first-order conditions that define a Bertrand equilibrium with the infringement for the implied marginal costs. The simulation is completed by solving the first-order conditions without the infringement for the Bertrand equilibrium absent of the infringement.

The published court opinions in *Mor-Flo* provide neither estimated demand elasticities nor data from which to estimate them, but the district court found facts about State's sales and profits that indicate that its average price-cost margin during the infringement period was 0.1635. This margin can serve in place of one of the two demand parameters. For any given value of ε, there is a unique β that implies the price-cost margin found by the court (given the share and price assumptions immediately below). We consider three different values of ε (−0.5, −1, and −2) and the corresponding values of β (11.948, 9.484, and 8.7746) that imply the price-cost margin found by the court. Because less energy efficient water heaters and electric powered water heaters were available, there are strong reasons to suspect that the demand for energy-efficient, residential gas water heaters was elastic,[20] and therefore that the most relevant of the three elasticities of demand probably is −2.

The court opinions indicate that there were five firms in the industry and that State had a 40% share. Lacking information on the remaining shares, we assume they were equal. Financial data in the opinion indicate that the patentee's price was somewhat greater than that of the infringer, and we normalize the patentee's price to 1 and the rival's prices to 0.9.

Since we have normalized both price and output (by using shares), it is useful to consider the calculated damages relative to some convenient benchmark, and we use State's profits with infringement. Table 1 presents calculated damages assuming just a single rival was infringing. It also presents an estimate of damages based on the application of the market-share rule. In general, the market-share rule can over- or understate actual damages, and that is also the result here, depending on the aggregate elasticity of demand. We believe that the most elastic demand assumption

Table 1. Damages as a percentage of profits with infringement in a hypothetical case based on *Mor-Flo*

Method of computation	Elasticity of demand		
	−0.5	−1	−2
Simulation	19.7	17.4	13.8
Market-share rule	17.7	17.7	17.7

is the most realistic, and for that assumption, the market-share rule overstates the simulated damages by 28%.

Even when the logic of the market-share rule is correct, as it is by construction with logit demand, the effects on the patentee's profits of price erosion, quantity accretion, and variety enhancement can be significant. With differentiated products, failing to account for the variety enhancement effect easily can be the greatest source of error. If it is possible to estimate the relevant demand parameters or even make educated guesses about them, simulation is likely to offer an improvement over the application of the market-share rule.

7. Generalizing the Market-Share Rule

The IIA assumption implicit in the market share rule is likely to be an inadequate description of consumer preferences in many particular cases. When the IIA property is not a reasonable characterization of preferences, the sort of damage simulation described in the previous section can be undertaken using a somewhat more flexible demand system. In merger simulation, four demand systems have been used—linear, isoelastic, AIDS, and nested logit.[21] Other demand systems that could be used are the mixed logit system (Berry *et al.*, 1995; Brownstone and Train, 1999). We illustrate only the nested logit model.

The leading case declining to apply the market-share rule is *BIC Leisure Products, Inc. v. Windsurfing International, Inc.*[22] Windsurfing held a patent on sailboards that perhaps encompassed all boards in the market. The patent was licensed to many competitors, generally at a 7.5% royalty. In addition, Windsurfing manufactured sailboards that embodied its patent. The district court held that it was reasonable to assume that, had BIC not been in the market, its sales would have been distributed among the other competitors in proportion to their market shares; hence, the district court awarded lost profits by applying the market-share rule. The court of appeals reversed, holding that Windsurfing did not establish causation, in light of substantial differences in product design and price. Damages were limited to reasonable royalties—the 7.5% Windsurfing was charging others.

The court of appeals probably was right to reject the IIA assumption in this case and to reverse the award premised on the market-share rule. Rejecting the IIA assumption, however, should not mean that damages are limited to reasonable royalties. To the extent the court found that causation was unproved, we think the court erred by failing to focus on whether the infringement caused Windsurfing to lose profits by selling less or at a lower price. To the extent the court simply could not fathom how to assess lost profits, we offer a simulation method for doing so.

Because the published opinions contain neither demand elasticity estimates nor data from with we can estimate the elasticities, we are unable to simulate damages in this case in quite the way described above. We must make some simplifying assumptions, including a parsimonious specification for demand; hence, we assume a nested logit model in which all of the nests have the same nest parameter, δ. With S nests and the set of products contained in nest s denoted by N_s, the probability of choice i in nest j is

$$\pi_i = \frac{\exp((\alpha_i - \beta p_i)\delta^{-1}) \, [\Sigma_{\kappa \varepsilon N_j} \exp((\alpha_\kappa - \beta p_k)\, \delta^{-1})]^{\delta-1}}{\exp(\alpha_0) + \sum_{s=1}^{S} [\Sigma_{\iota \varepsilon N_s} \exp((\alpha_\iota - \beta p_\iota)\, \delta^{-1})]^{\delta}}$$

Table 2. Court and simulation damages estimates in *BIC Leisure* (thousand dollars)

	Licensee sales	Patentee sales	Total
Original award	750	2102	2852
Simulated damages, $\delta = 1$	376	1115	1491
Simulated damages, $\delta = 0.8$	403	890	1293
Royalty only award			1079

We divide the various competing products into four nests on the basis of their prices.[23] We do not know from the published court opinions how strong the nests should be, and arbitrarily assume a nest parameter of 0.8. This makes the nests strong enough to matter while allowing significant competition with products outside a nest. We assume that the aggregate elasticity of demand for all sailboards is −1 in the with-infringement equilibrium. Finally, we calibrate the logit β to imply a marginal cost for the patentee equal to that found by the district court. In doing this calibration and in computing the damages, we account for the fact that the patentee was earning a 7.5% royalty on licensed sales by the non-infringing rivals.

Table 2 presents four damage estimates: the original award made by the district court based on the market-share rule, our simulation estimate, a second simulation estimate imposing the IIA assumption (i.e. with a nest parameter of 1), and the royalty-only award made by the court of appeals. The damages are separated into components relating to the patentee's own sales and the sales of licensees, although the royalty-only award fits neither category. Perhaps most significantly, the original award, based on the market-share rule, is nearly double the simulation estimate imposing the IIA assumption. Simulation results in a lower damage estimate than the market-share rule because simulation properly accounts for quantity accretion and variety enhancement.

8. Lost Profits with Non-infringing Substitutes: Auctions

Although differentiated consumer products were involved in leading cases on lost profits from patent infringement, the simulation approach has a more general application. Simulation can be usefully employed to assess lost profits from infringement whenever the relevant competitive interaction can be modeled well using a static oligopoly model with a unique equilibrium. As a final illustration of the potential applications of damage simulation, we posit that the patentee, the infringer, and others compete to sell through an auction mechanism. For simplicity, we consider a private-values, second-price auction[24]—a procurement auction that awards a single contract to the low bidder.[25]

In a second-price auction (with a purchasing auctioneer), the low bidder wins the auction and sells at a price equal to the second-lowest bid. As explained by Vickrey (1961), each bidder's dominant strategy is to bid its cost, because one's own bid determines whether that one wins the auction but does not affect the price paid. In the private-values context, a second-price auction is equivalent to an oral auction.

The patentee is damaged by the infringement in any procurement in which the patentee and the infringer are the two lowest bidders. In such instances, there is necessarily price erosion and possibly sales diversion as well. There is sales diversion if the infringer is the low bidder and the patentee is the next-lowest bidder. In that event, the patentee would have won the auction but for the infringement. There is price erosion in that event as well, because the infringer's low bid causes the price-determining, second-lowest bid to be the patentee's bid instead of the bid just above it. There is also price erosion if the patentee is the low bidder and the infringer is the next-lowest bidder. In that event, the infringer's bid causes the price-determining, second-lowest bid to be the infringer's bid instead of the bid just above it. Computing damages, thus, involves a determination of how frequently the patentee and infringer are the two lowest bidders and what the price would have been absent of the infringer's bid.

Computing damages as *actual* lost profits is simple with complete data on each auction, including losing bids. Such data, however, are often unavailable, since only winning bids may be recorded. Data on just winning bids is sufficient to compute damages based on the patentee's simulated *expected* lost profits. To do this, one first calibrates a tractable model to the with-infringement equilibrium, then determines the expected equilibrium after eliminating the infringer as a bidder. Froeb *et al.* (1999a,b) have identified a class of auction models that can be characterized by market shares (how frequently each bidder wins) and the distribution of the minimum cost (over all bidders).[26] Following Waehrer and Perry (1999), they assume that bidders take independent cost draws from cost distributions that differ across bidders but are all 'power-related.' What this means is that bidder i's cumulative cost distribution, $1-G_i(x)$, is related to some parent distribution, $F(x)$, in the following manner:

$$G_i(x) = [1 - F(x)^{r_i}].$$

This form of cost asymmetry would result if bidders drew from the same cost distribution but differed in the number of independent draws, r_i, they took from it. The greater r_i, the lower bidder i's expected cost and the greater its expected share of wins.

If s_i is the probability that bidder i wins the auction or the 'share' of bidder i, and $1 - G_{\min}(x)$ is the cumulative distribution of minimum cost over all bidders, the family of distributions can be characterized as follows:

$$G_i(x) = [G_{\min}(x)]^{r_i}], \; G_{\min}(x) = [1 - F(x)]^{r_{\min}}, \; r_{\min} = \sum_{i=1}^{n} r_i, \; s_i = r_i/r_{\min}.$$

And letting

$$r_{-i} = r_{\min} - r_i, \; G_{-i}(x) = [1 - F(x)]^{r_{-i}},$$

the closed-form expression for the distribution of the winning price (which is the second-lowest bid), p_i, conditional on bidder i having the lowest cost draw is

$$F_{p_i}(x) = \frac{1}{s_i}[1 - G_{-i}(x)] + \left[1 - \frac{1}{s_i}\right][1 - G_{\min}(x)].$$

Table 3. Simulated damages in a hypothetical auction setting

	Patentee	Infringer	Third bidder
With infringement			
expected share	0.3	0.5	0.2
expected winning bid	109.3	110.8	108.7
expected profit	2.8	5.4	1.7
But for infringement			
expected share	0.6		0.4
expected winning bid	117.3		115.4
expected profit	7.1		4.0

The expected value of bidder *i*'s winning bid is

$$E(p_i) = \frac{1}{s_i}\mu(r_{-i}) + \left[1 - \frac{1}{s_i}\right]\mu(r_{\min}),$$

where $\mu(r)$ is the mean of a random variable with cumulative distribution $1-[1-F(x)]^r$. Both expressions are weighted averages, with weights based on the bidder's probability of winning, s_i. The quantities averaged are associated with the minimum cost for bidders other than *i* and for all bidders. Expected bids are decreasing in the respective shares because larger firms expect to win at higher prices than smaller firms. Losing bidders set the price, and low-cost, large-share bidders expect to win while bidding less aggressively than higher-cost rivals.

For a simple illustrative simulation, we assume that the distribution of minimum cost has an extreme-value type III distribution. This distribution gives rise to logit demand (Werden and Froeb, 1994) and logit auction models (Froeb *et al.* 1999a, b) and is relatively easy to estimate (Brannman and Froeb, forthcoming). We assume that the cost distribution for the minimum cost over all bidders has a mean of 100 and a standard deviation of 10. The expected cost of the winner (which has the minimum cost), thus, is also 100. We assume that there are three bidders with infringement—the patentee, the infringer, and one non-infringing competitor—with shares as indicated in Table 3. The table also displays the shares, expected winning bids, and profits of the three bidders, both with and without infringement.[27]

The infringer's share is redistributed to the remaining bidders in proportion to their shares with infringement. This again is the IIA property, which underlays the courts' market-share rule. The IIA property in this model stems from the assumption of power-related cost distributions, and it can arise with some other assumptions as well. The IIA property does not hold if there is correlation among some bidder's cost draws (see Bajari, 1996). One way to introduce such correlation is through the addition of one or more nests. Introducing a positive correlation in two bidders's cost draws increases the frequency with which they are the two lowest bidders. A positive correlation between the costs of the patentee and infringer increases expected lost profits.

9. Conclusion

We offer simulation as a methodology for assessing lost profits damages in patent damages cases. Simulation provides reasonable lost profits estimates in cases with and without non-infringing substitutes, and accounts for potentially significant effects that the courts have often ignored. Simulation accounts for price erosion and the corresponding quantity accretion, and it accounts for the effects infringement can have on total market quantity as a result of enhancing product variety.

Simulation should help to refocus the causation analysis in patent infringement cases. When an infringer competes with the patentee, that competition results in lost profits for the patentee, and proof of significant competition ought to be sufficient to establish the causation necessary for recovery of lost profits. What remains is for the patentee to make a reasonable estimate of the lost profits, and simulation provides a methodology for doing so.

Simulation, of course, can provide only a reasonable estimate. There will always be a potential for error due to the fact that the assumptions on which the simulations are based are never precisely right, and there is statistical variation in any estimation of demand elasticities. But the tendency of the courts to resolve uncertainty against the infringer should assure that simulated damage estimates are not considered speculative. Moreover, nothing in the case law prohibits refinements in methods of assessing damages.

Notes

1. 35 U.S.C. § 284.
2. King Instruments Corp. v. Perego, 65 F.3d 941, 952, 36 U.S.P.Q.2d (BNA) 1129, 1137 (Fed. Cir. 1995).
3. Our proposal is not entirely novel. Authors Froeb and Langenfeld used a version of it in consulting work on an actual patent infringement case, as did the opposing expert, David Evans. An exercise based on that case is available on-line at http://www.antitrust.org/simulation.html. The work of many other economists on patent damages cases accounts for important economic forces implicit in our suggested analysis.
4. For discussions of the rationale for, and details of, merger simulation, see Crooke *et al.* (1999), Werden and Froeb (1996), and Werden (1997a, b, c).
5. Werden *et al.* (1999b) discuss the case law is more detail.
6. Amstar Corp. v. Environtech Corp., 823 F.2d 1538, 1543, 3 U.S.P.Q.2d (BNA) 1412, 1415 (Fed. Cir. 1987).
7. Standard Havens Products, Inc. v. Gencor Industries, Inc., 953 F.2d 1360, 1373, 21 U.S.P.Q.2d (BNA) 1321, 1331 (Fed. Cir. 1991).
8. Radio Steel & Manufacturing Co. v. MTD Products, Inc., 788 F.2d 1554, 229 U.S.P.Q. (BNA) 431 (Fed. Cir. 1986). The features cited were the ability to be shipped unassembled and the absence of a brace along the rear of the legs.
9. BIC Leisure Products, Inc. v. Windsurfing International, Inc., 1 F.3d 1214, 1219, 27 U.S.P.Q. 2d (BNA) 1671, 1675 (Fed. Cir. 1993).
10. Kerwit Medical Products, Inc. v. N & H Instruments, Inc., 224 U.S.P.Q. 679, 688 (N.D. Tex. 1984).
11. Werden (1983, 1992, 1993, 1998) provides detailed discussions of the approaches to antitrust market delineation of the Merger Guidelines and the case law.
12. The importation of antitrust market delineation, however, should produce satisfactory results when damage simulation is impractical.
13. This point has been noted by Addanki (1995: 3), Merges (1997: 1069–74), Rapp and Beutel (1996; 124–27), Schilicher (1997: § 9.05[2][a]), and Werden *et al.* (1999a).
14. Demand estimation is common in antitrust cases and has been done in some infringement cases. The sort of price variation that is essential in accurately estimating demand elasticities may be more likely

to exist in the infringement cases because it results from the infringement itself. Simulation can also be performed using elasticities inferred from various economic data or intuited from non-quantitative evidence.

15. As indicated by research on merger simulations (Crooke *et al.*, 1999), damages may be substantially greater if the demand curve is assumed to be isoelastic or AIDS than if it is assumed to be linear or logit.
16. When marginal costs are inferred, accounting estimates are useful cross checks.
17. It may be possible to estimate marginal cost functions when marginal costs are not constant.
18. 883 F.2d 1573, 12 U.S.P.Q.2d (BNA) 1026 (Fed. Cir 1989), *aff'g* 8 U S.P.Q.2d (BNA) 1971 (E.D. Tenn. 1988).
19. Mor-Flo argued that State's national market share was irrelevant, since Mor-Flo sold mostly in California, where State had a very small share. The courts rejected this argument on a rather unclear basis, but it seems likely that Mor-Flo had a valid point. Water heaters are bulky and relatively expensive to transport. Consequently, manufacturers tend to have higher shares of sales nearer to their manufacturing plants. Mor-Flo had a plant in California, while State did not have a plant anywhere in the western United States. On the other hand, State and Mor-Flo both had plants in Tennessee.
20. Shortly after the period of the alleged infringement, the Department of Justice challenged the merger of two firms in the industry. The relevant market alleged in the complaint was all residential water heaters, suggesting that neither energy-efficient water heaters nor gas water heaters were a distinct relevant market for antitrust purposes.
21. These are discussed and compared by Crooke *et al.* (1999). Merger simulation with linear and isoelastic demand is discussed by Werden (1997c), and merger simulation with AIDS demand is discussed by Hausman and Leonard (1997).
22. 1 F 3d 1214, 27 U.S.P.Q.2d 1671 (Fed. Cir. 1993), *reversing* 761 F Supp. 1032, 19 U.S.P.Q. 2d (BNA) 1922 (S.D.N.Y. 1991).
23. Additional details on the assumptions of this simulation and its results are provided by Werden *et al.* (1999b).
24. As stated at the outset, damages simulation is an adaptation of merger simulation. For discussions about merger simulation in first-price or sealed bid auctions, see Tschantz *et al.* (1997) and Dalkir *et al.* (2000).
25. We assume that the auctioneer does not set a reserve price and the contract is always let. Relaxing this assumption gives the auctioneer some bargaining power. It is straightforward in that case to compute optimal reserve prices.
26. For high-bid auctions, this class of distributions can be characterized in terms of the shares and the distribution of the maximum value
27. Eliminating the infringer changes the distribution of the minimum cost over all bidders, increasing its mean by 5.4.

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